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BIOGRAPHY.

PAFNUTIJ LVOVITSCH TCHEBYCHEV.

BY DR. GEORGE BRUCE HALSTED.

OF Russian mathematicians, second only to Lobachevsky should be ranked Pafnutij Lvovitsch Tchebychev. Born in Russia in 1821 and formerly professor at the University of St. Petersburg, he reached deservedly the very highest scientific honors, being privy councillor, the representative of applied mathematics in the Imperial Academy of St. Petersburg, in 1860 made member of the famous Section I.—Geometrie, of the French *Academie des Sciences*, and afterward *Associe Etranger*, the highest honor attainable by a foreigner.

His best known work is the justly celebrated *Memoire Sur les Nombres Premiers*, (Academie imperiale de Saint-Petersbourg, 1850), where he established the existance of limits within which the sum of the logarithms of the primes inferior to a given number must be comprised. This memoir is given in *Liouville's Journal*, 1852, pp. 366-390.

Sylvester afterward contracted Tchebychev's limits; but the original paper remains highly remarkable, especially as it depends on very elementary considerations. In this respect it is in striking contrast to the equally marvelous paper of the lamented Riemann, *Ueber die Anzahl der Primzahlen unter einer gegebenen Groesse* [einer Untersuchung ueber die Haeufigkeit der Primzahlen], presented to the Berlin *Academie* in 1859.

Tchebychev had in 1848 presented a paper with this very title to the St. Petersburg Academie: Sur la totalite des nombres premiers inferieurs a une limite donnee. (Given in Liouville's Journal, 1852, pp. 341-365.) Riemann speaks of the interest long bestowed on this subject by Gauss and Dirichlet, but makes no mention of Tchebychev. However Sylvester speaks of "his usual success in overcoming difficulties insuperable to the rest of the world."

But though best known for his work in the most abstract part of mathematics, in reality Tchebychev was of an eminently practical turn of mind. Thus it was his work "*Theorie des Mecanismes connus sous le nom de parallelogrammes* (Memoires des savants etrangers, Tom. VII.) which led him to the elaborate dissertation *Sur les questions de Minima qui se rattachent a la representation approximative des fonctions*, 91 quarto pages in Memoires de l'Academie Imperiale des sciences de Saint Petersbourg, 1858. While the variable x remains in the vicinity of one same value we can represent with the greatest possible approximation any function $f(x)$, of given form, by the principles of the differential calculus. But this is not the case if the variable x is only required to remain within limits more or less extended. The essentially different methods demanded by this case, which is just the one met in practice, are developed in this memoir. The same line of thought led to his connection with a subject which has since found a place even in elementary text-books, namely rectilineal motion by linkage. He invented a three-bar linkage, which is called Tchebychev's parallel motion, and gives an extraordinarily close approximation to exact rectilineal motion; so much so that in a piece of apparatus exhibited by him in the London Loan Collection of Scientific Apparatus, a plane supported on a combination of two of his parallel motion linkages seemed to have a strictly horizontal movement, though its variation was double that of the tracer in the simple parallel motion.

Tchebychev long occupied himself with attempting to solve the problem of producing exact rectilineal motion by linkage, until he became convinced that it was impossible and even strove long to find a proof of that impossibility. What must have been his astonishment then, when a freshman student of his own class, named Lipkin, showed him the long-sought conversion of circular into straight motion.

Tchebychev brought Lipkin's name before the Russian government, and secured for him a substantial reward for his supposed original discovery.

And perhaps it was independent, but it had been found several years previously by a French lieutenant of engineers, Peaucellier, and first published by him in the form of a question in the '*Annales de Mathematique* in 1864. When Tchebychev was on a visit to London, Sylvester inquired after the progress of his proof of the impossibility of exact parallel motion, when the Russian announced its double discovery and made a drawing of the cell and mounting. This Sylvester happened to show to Manuel Garcia, inventor of the laryngoscope, and the next day received from him a model constructed of pieces of wood fastened with nails as pivots, which, rough as it was, worked perfectly. Sylvester exhibited this to the Philosophical Club of the Royal Society and in the Athenaeum Club, where it delighted Sir Wm. Thomson, now Lord Kelvin, and led to the extraordinary lecture *On recent Discoveries in Mechanical Conversion of Motion*, delivered by Sylvester before the Royal Institution on January 23, 1874. This in turn led to Kempe's remarkable developement of the subject, and to Hart's discovery of a five-bar linkage which does the same work as Peaucellier's of seven. Henceforth Peaucellier's Cell and Hart's Contraparallelogram will take their place in our text-books of geometry, and straight lines can be drawn without begging the question by assuming first a straight edge or ruler as does Euclid. Thus Kempe's charming book, *How to draw a straight line*, is a direct outcome of Tchebychev's sketch for Sylvester, to whom, in parting, he used the characteristic words: "Take to kinematics, it will repay you: it is more fecund than geometry; it adds a fourth dimension to space." As might perhaps have been expected, the immortal Lobachevsky found in his compatriot a devoted admirer. Not only was Tchebychev an active member of the Committee of the Lobachevsky-fund, but he took the deepest interest in all connected with the spread of the profound ideas typified in the Non-Euclidean geometry.

Knowing this, Vasiliev in his last letter asked that a copy of my translation of his Address on Lobachevsky be forwarded to the great man. His active participation in scientific assemblies is also worthy of note; for example at the 'Congres de l'Association francaise pour l' avancement des sciences, a Lyon' he read two interesting papers, *Sur les valeurs limites des integrales*, and *Sur les quadratures*, both afterwards published in *Liouville's Journal*.

LAGRANGE'S GENERALIZED EQUATIONS OF MOTION.

By F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Lagrange's celebrated equations of motion, as given in his *Mecanique Analytique*, consist of *a transformation* from Cartesian to generalized co-ordinates, *of the indeterminate equation of motion.* In order to avoid the unnecessary complication incident to the introduction of such indeterminate variations, as δx, δy, δz, etc., in the derivation of these equations of motion, we must have recourse to the Cartesian equations of the unconstrained motion of particles.

Assume $(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)$ as the rectilineal-rectangular co-ordinates of the material particles $m_1, \ldots, m_n$; also, assume $(X_1, Y_1, Z_1), \ldots,$ (X_n, Y_n, Z_n) as the force components of the particles. Make $(\psi_1, \phi_1, \theta_1), \ldots,$ $(\psi_n, \phi_n, \theta_n)$ the generalized co-ordinates of the *respective* particles at any instant of time; that is, let ψ_1, ϕ_1, θ_1, be regarded as determinate functions of x_1, y_1, z_1, respectively,—or vice versa. This reciprocal determinateness is to be a characteristic of all the material particles. The velocity-components in